Biological tissue mechanics with fibres modelled as 1D *Cosserat* Continua.

Applications to cardiac tissue.

Kevin Sack[1,4], Sebastian Skatulla[1,4], Carlos Sansour[5] & Thomas Franz[1,2,3]

[1]Centre for Research in Computational and Applied Mechanics, University of Cape Town, Cape Town, South Africa; [2]Department of Human Biology, University of Cape Town, South Africa; [3]Centre for High Performance Computing, Rosebank, South Africa:

[4]Computational Continuum Mechanics Group, Department of Civil Engineering, University of Cape Town, Cape Town, South Africa; [5]Division of Materials, Mechanics, and Structures, The University of Nottingham, United Kingdom

Introduction

Cardiovascular disease is the single leading cause of death [1, 2]. Recent advancements in numerical methods and the proliferation of inexpensive high performance computing power has enabled more sophisticated simulation tools that allow for greater insight into cardiovascular function and disease that can assist the development of modern therapies. Cardiovascular tissue is a complex heterogeneous material with a significant hierarchical micro-structure that influences the material on a macro scale [3, 4, 5, 6]. Mathematically described as a non-linear, hyperelastic material with orthotropic properties arising from the micro-structural influence of the myocyte fibres embedded within the cytoskeleton.

The need to place a larger emphasis on the micro-structure is expressed heavily in the literature [7, 8, 9]. Increasing attempts to include the micro-structural influences have emerged attempting to better capture the complex material response. The successful inclusion of generalized continua approaches, including *Cosserat* models, in the application to hard biological tissue [5, 10, 11, 12], has proved successful in capturing more realistic material responses. However soft biological tissue remains significantly under-investigated with respect to micro continuum theories.

This research builds on the work of Sansour and Skatulla [13] and extends its application to soft biological tissue.

Mathematical model

The complex cardiac tissue components are considered under two main categories:

1. The fibrous structure: composed of a bundle of myocytes, defining the anisotropy of the myocardium.
2. The complementary connective tissue: made up from remaining components of the extracellular matrix (ECM).

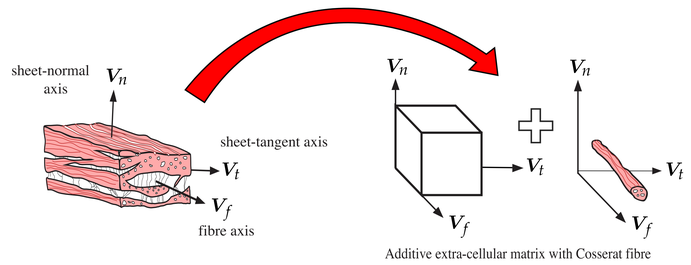


Figure 1: The cardiac specimen is modelled by considering contributions from a three-dimensional ECM and a fibre bundle. Image of cardiac specimen reproduced from [8].

Unlike classical approaches, the strain energy function depends on two strain measures which are the *Cosserat* strain measures **U** and **K**, respectively:

$$\psi(\mathbf{U},\mathbf{K}) = \frac{1}{2}A\left(e^{BQ}-1\right) + A_{comp}\left(J\ln(J) - J + 1\right) \tag{1}$$

The stress scaling coefficients are given by A, A_{comp} and B; J is the *Jacobian* and Q is defined by

$$Q = a_1 U_{ff}^2 + a_2(U_{tf}^2 + U_{nf}^2) + a_2\frac{I_1}{A_f}K_{ff}^2 + a_1\frac{I_2}{A_f}(K_{tf}^2 + K_{nf}^2)\ \ldots \qquad \text{Fibre contributions}$$
$$+\ \ldots\ b_1(U_{tt}^2 + U_{nn}^2) + b_2(U_{ft}^2 + U_{fn}^2 + U_{nt}^2 + U_{tn}^2) \qquad \text{ECM contributions}$$

The material parameters a_i and b_i govern the material anisotropy. Considering transversely isotropic material behaviour, we introduce geometrical properties of the *Cosserat* rod linked to the characteristic length l (i.e. the micro structure):

$$\underbrace{A_f = \tfrac{\pi l^2}{4},}_{\text{Cross sectional area,}} \qquad \underbrace{I_1 = \tfrac{\pi l^4}{32}}_{2^{nd}\text{ Moment of area (torsion)}} \qquad \underbrace{I_2 = \tfrac{\pi l^4}{64}.}_{2^{nd}\text{ Moment of area (bending)}} \tag{2}$$

Variational principle

This allows us to express the variation of the internal potential of the body with reference density ρ_0 as:

$$\delta\Psi = \int_{\mathcal{B}} \left\{\rho_0\frac{\partial\psi(\mathbf{U},\mathbf{K})}{\partial\mathbf{U}} : \delta\mathbf{U} + \rho_0\frac{\partial\psi(\mathbf{U},\mathbf{K})}{\partial\mathbf{K}} : \delta\mathbf{K}\right\} dV = \int_{\mathcal{B}} \{\boldsymbol{n} : \delta\mathbf{U} + \boldsymbol{m} : \delta\mathbf{K}\}\, dV \tag{3}$$

where $\boldsymbol{n}$, the force stress tensor, and $\boldsymbol{m}$, the couple stress tensor, are work conjugate to **U** and **K**, respectively. In the static case with only mechanical considerations, the first law of thermodynamics provides the variational statement

$$\delta\Psi - \mathcal{W}_{ext} = 0\,, \tag{4}$$

Calibration

All computer modelling is done using the in-house modelling software SESKA. SESKA is a numerical modelling software, based on the element free Galerkin method (EFGM). Calibration employs a two stage approach that utilizes experimental shear data sets [14] and passive filling experimental data [15, 16, 17].

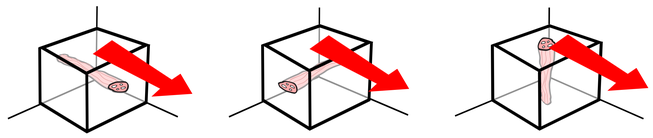


Figure 2: Modes of shear for transverse isotropic myocardium: (a) Transverse plane shifted towards the fibre direction, (b) Transverse plane shifted towards the other transverse plane and lastly (c) Fibre plane shifted towards a transverse plane.

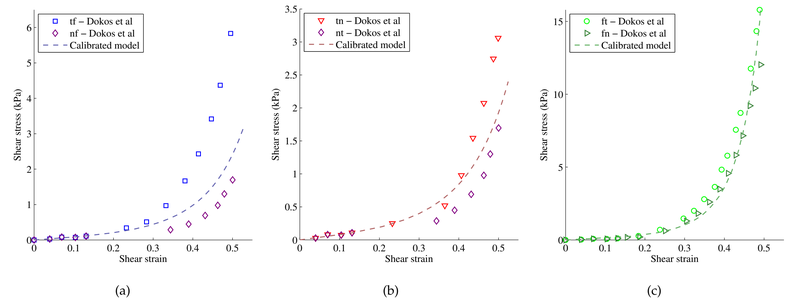


Figure 3: Material response of calibrated nonlinear *Cosserat* fibre model (dashed lines) alongside the combined data sets [14]. cases (a)-(c) the same as Fig. 2.

The resulting calibrated model results are shown in Fig. 4. The model was able to reproduce the experimental mean with great accuracy, resulting in a calibrated fit with $R^2 = 0.9988$.

Parameter	Symbol	Value
Principle fibre modulus	a_1	52.380
Shear fibre modulus	a_2	28.090
Principle matrix modulus	b_1	18.112
Shear matrix modulus	b_2	16.480
Characteristic length	l	0.6622
Stress Scaling 1	A	0.445
Stress Scaling 2	B	0.138
Incompressibility	A_{comp}	50.0

Table 1: Material properties for nonlinear *Cosserat* constitutive law, after the two stage calibration using a levenberg marquad optimization algorithm.

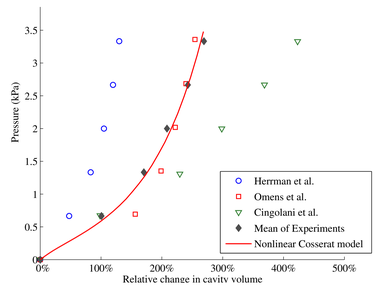


Figure 4: Passive material response of an ellipsoid model of the rat left ventricle. The *Cosserat* fibre model (627 particles) is calibrated to the mean of [15, 16, 17].

Quantifying the micro-structural influence

Cosserat fibre model improves on the classical elasticity formulation by

- Including higher order strains, i.e torsion and bending in the constitutive model.
- Accounting for the relative motion of of neighbouring material points.
- Providing a more realistic material description.
- Connecting the micro-structural influence to macroscopic material behaviour in a meaningful way.

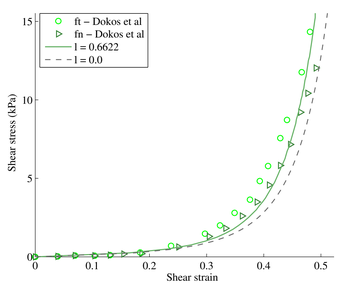
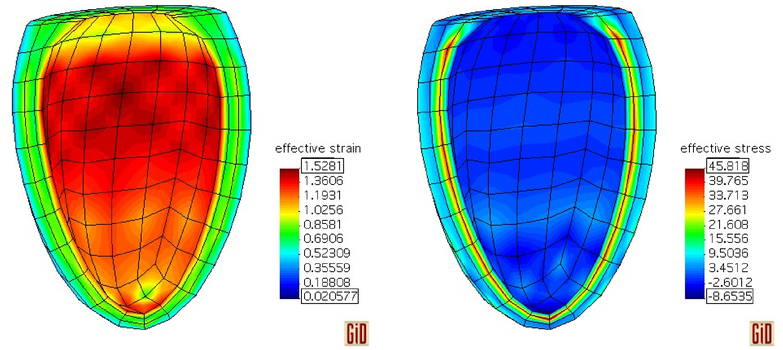


Figure 5: Material response of calibrated nonlinear *Cosserat* fibre model alongside the case for $l = 0.0$.

Applications

Initial investigations into passive material response in the rat left ventricle were performed with successful qualitative outcomes.



(a) Strain (b) Stress (kPa)

Figure 6: Left ventricle simulations at the end of diastole. Solutions of stress and strain are plotted on half of the left ventricle for the rat.

- Deformation features dilation, mild torsion and extension [18]. ✓
- Peak strains are experienced at the endocardium wall [19, 20]. ✓
- Peak stresses occurs between the endocardium and the midwall [20, 21]. ✓

Conclusions

The *Cosserat* fibre continuum description for cardiac tissue has been successfully developed and calibrated to describe its passive material behaviour in a more detailed and micro-structurally motivated fashion. The defining feature of the model is the inclusion of micro-structural deformation modes referring to twist and flexure of fibre bundles via one-dimensional *Cosserat* continua. The micro structural influence towards the macroscopical material behaviour is controlled by the characteristic length l which enters the constitutive law through geometrical properties of the fibre bundle.

Without additional experimental data, a unique value for l cannot be determined with complete confidence. As no such suitable data exists, l is calibrated to achieve the lowest possible residual in the optimization routine. However, it is demonstrated that the internal length associated with the change of curvature strains significantly influence the material behaviour on a micro-structural level.

Acknowledgement

This research has been supported by the Centre for High Performance Computing (CHPC) and the National Research Foundation (NRF) of South Africa.

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